Characterization of Microwave Variable Capacitance Diodes*

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Summary—This paper will describe the electrical characterization of microwave variable capacitance diodes. The importance of some of the diode parameters is discussed from the application point of view, and suitable measurement techniques for these parameters are described, together with actual measurement data on some diodes.

First, a general four-terminal transformation method is used, and some approximations lead to a fairly easy and accurate method of studying device characteristics. A resonant-cavity method is also considered, and it is explained under what condition it leads to a very simple test of the diode Q.

Finally, a method is presented which is based upon modifications of the Weissfloch canonical network. These simplifications can be used to get an easy interpretation of the junction impedance or the diode Q.

I. Introduction

Microwave variable capacitance diodes have recently been widely used in low-noise systems, microwave computers and harmonic generators; however, since the feasibility of using the diodes in these applications is fairly new, the problem of the characterization may be unfamiliar to the user. It should be pointed out that since we, in some instances, are touching the impedance measurement limit of the microwave instruments, the problem is not a simple one.

The techniques described in this paper should be sufficient for a device designer to evaluate the device parameters in the research laboratory as well as in production. The circuit designer will also find the measured parameters sufficient for the calculation of performance in many applications.

The first part of this paper describes the importance of the different diode parameters and the diode package. Then, together with some curves showing the actual behavior of a diode, different ways of measuring these characteristics at low as well as microwave frequencies are described. The transition capacitance which is one key parameter in all the applications, the capacitance nonlinearity coefficient, as well as the dc volt-ampere characteristic, and the breakdown voltage of a variable capacitance diode, are discussed.

Considerable attention is focused upon the measurement of the diode loss represented by the bulk series resistance of the wafer, since the losses are the most undesirable parameter in circuit applications. Three different approaches are considered:

a) a general four-terminal transformation method,
b) a resonant-cavity method,
c) a simplified canonical-network transformation method.

Some of the pertinent practical aspects of each of these methods are discussed.

II. Circuit Considerations

The most satisfactory electrical performance should be the guide in selecting the parameters useful in optimizing the physical structure of the device.

The capability of the parametric diodes is partially determined by the noise figure which can be achieved in microwave communication or radar and radio astronomy systems, by the transient response and input power requirements to establish oscillations in applications in binary computers, and by the efficiency in generating harmonics in harmonic generators.

First of all, these applications suggest low diode losses. The most common and convenient measure of the loss in a circuit element is the quality factor Q which measures the ratio of energy stored to energy lost per cycle. For an equivalent circuit of the diode consisting of the diode transition capacitance C_T in series with the bulk series resistance R_s, the Q is defined as

$$Q = \frac{1}{\omega R_s C_T} = \frac{f_c}{f}, \tag{1}$$

where f_c is defined as the cutoff frequency of the diode. The transition capacitance should be small for microwave impedance matching purposes when the diode is used in low-noise amplifiers.

The nonlinear capacitance coefficient $(\partial c/\partial v)\ v = v_{d_0}$ should be high for good noise figure, for high-harmonic generator efficiency, and for low-rise time in computer applications. The nonlinearity coefficient is also important from pump-voltage considerations. A higher $(\partial c/\partial v)\ v = v_{d_0}$ minimizes the necessary pump power. This is an important point in a frequency range where high power sources are not commercially available and especially in computer circuits where the same pump oscillator is driving several subharmonic generators.

The I-V characteristics of the diode should show a reverse breakdown voltage of at least -5 volts defined

* Received by the PGMTT, July 5, 1960; revised manuscript received, August 19, 1960. This paper was presented at the 1960 IRE Symp. of the PGMTT, San Diego, Calif.
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for 10 μa leakage current. Several volts of pumping oscillator swing are sometimes required for amplifier operation. A low breakdown voltage might cause burnout of the diode and shot noise effect. For harmonic generator application, it is especially desirable to have breakdown voltages of around -25 volts.

The range over which microwave diodes will operate satisfactorily is to a great extent determined by the electrical properties and the mechanical dimensions of the diode package. What is wanted is essentially as little package interaction as possible in different circuits. The diode package, therefore, should have low losses at the highest microwave frequencies, low stray capacitance and inductance, and mechanical dimensions fitted to coaxial, waveguide and strip-line circuits. A cylindrical package with quartz as insulating material and dimensions such as an 80-mil diameter and a 50-mil height with a stray capacitance of around 0.1 $\mu\mu$f or less and an inductance measured at UHF of 1 mμh, will meet most present requirements.

III. Low Frequency and DC Characterization

The transition capacitance C_T is a key parameter in all the applications and therefore should be measured accurately. A satisfactory frequency for measurement is 100 kc, because inductance effects are negligible in a high grade unit. A simple bridge circuit with an oscillator and an amplifier for null detection can be used. A suitable bridge for this purpose is the Boonton Electronics Capacitance Bridge 74CS4. A dc bias supply for measurements on diodes is built into the bridge. The signal voltage across the unknown capacitance should be less than 30 mv.

The transition capacitance at different frequencies as a function of voltage for a gold-bonded variable capacitance diode is plotted in Fig. 1. The measurement at 2 kMc was taken with a slotted line and agrees, within measurement error, with the 100 kc data. For the capacitance we can write:

$$C_T = C_0\left(1 - \frac{v}{\phi}\right)^{-\gamma}, \tag{2}$$

where ϕ is the contact potential and v is the applied voltage. C_0 is the zero-bias capacitance and is a function of the impurity doping level and the geometrical configuration and structure of the diode. The exponent γ is a constant which is equal to $\frac{1}{2}$ for the abrupt and $\frac{1}{3}$ for the linear-graded junction.

When expanding the capacitance in a Taylor series, we get:

$$C_T = (C_1)_{v=v_{dc}} + \left(\frac{\partial C_T}{\partial V}\right)_{v=v_{dc}} V_{ac} + 1/2\left(\frac{\partial^2 C_T}{\partial^2 V}\right)_{v=v_{dc}} V_{ac}^2 + \cdots \tag{3}$$

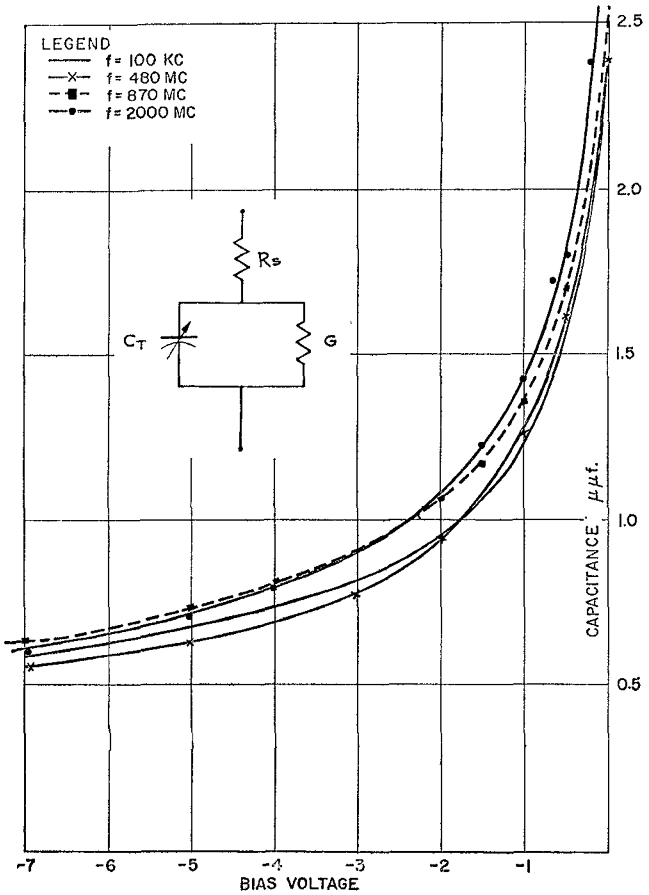


Fig. 1—Variation of junction capacitance with frequency; inset: the low frequency reverse-bias diode-equivalent circuit.

This is usually done in analytical work on amplifiers and subharmonic generators, and the first two terms are a fair enough approximation in these cases. The nonlinearity coefficient $(\partial C/\partial V)_{v=v_{dc}}$ will then be an important parameter for the pump power requirement as well as for the optimum value of the negative conductance. There is no need for a special measurement of the coefficient, however, since it can be determined from (2):

$$\frac{\partial C_T}{\partial v} = \frac{\gamma C_0}{\phi}\left(1 - \frac{v}{\phi}\right)^{-\gamma-1}. \tag{4}$$

In second-harmonic generators, the second-order term has to be included in the analysis. Then we have

$$1/2\left(\frac{\partial^2 C_T}{\partial v^2}\right) = 1/2\,\frac{C_0}{\phi^2}\gamma(\gamma+1)\left(1 - \frac{v}{\phi}\right)^{-\gamma-2}, \tag{5}$$

which is also determined by the diode structure, the contact potential and the bias voltage.

The volt-ampere characteristic of a Hughes HPA2800 diode is shown in Fig. 2. For a diode where the bulk resistance is neglected, we have

$$I = I_s(e^{qV/kT} - 1), \tag{6}$$

where $q/kT = 38.8$ volt^{-1} at 25°C. The measured value of the slope indicates a factor of 37 volt^{-1} and an exponential behavior which is in good agreement with theory. At very high currents, the slope indicates a bulk resistance of 2 ohms. This is a factor of 2 less than was obtained from microwave measurement. The discrepancy is most likely due to conductivity modulation. There is some correlation between the microwave series resistance and the series resistance measured by taking the slope of the dc curve at high forward currents. However, scatter diagrams show that a selection of devices by this method in the production line would not be possible.

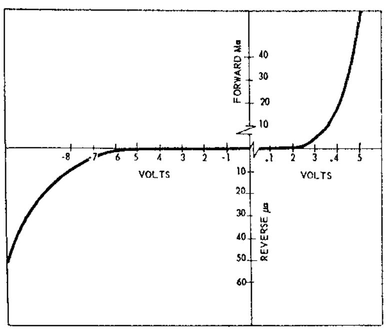


Fig. 2—The volt-ampere characteristic for a Hughes HPA2800 diode.

The diode conductance can be written

$$G = \frac{dI}{dV} = \frac{qI_s}{kT} \cdot e^{qV/kT}. \tag{7}$$

In the reverse direction, this conductance is in the order of micromhos. The reverse current in germanium, however, is mainly a surface leakage current. If a shot-noise contribution of 0.1 of the thermal noise contribution is tolerated when the diode is used in a low-noise amplifier, then we have for the leakage current I_c

$$I_c = \frac{0.14kT_0}{2eR_s} = 1.25 \text{ ma}, \tag{8}$$

where k = Boltzmann's constant $= 1.38 \times 10^{-23}$ per °K, $T_0 = 290$°K, $R_s = 4$ ohm, $e = 1.6 \times 10^{-19}$ coulombs. Thus, at room temperature, a fairly large leakage current can be tolerated if the Q of the diode was not degraded at the operating frequency. Since, however, the current is increasing very rapidly at the breakdown voltage, and the power dissipation in the diode increases the junction temperature, it would be wise to define the breakdown voltage, V_B, at 10 μa current and to select breakdown voltages of minimum -5 volts for diodes used in amplifiers and subharmonic generators. For harmonic generators where substantially higher input power may be required, a breakdown voltage as high as -25 volts is desirable.

The diode reverse-biased equivalent circuit at the low frequencies can be represented as in Fig. 1. The most convenient way to measure the series resistance R_s is at microwave frequencies where G is shunted out by C_T.

IV. Microwave Characterization

The problem of measuring with reasonable accuracy a high Q impedance consisting of say 1–2 ohms resistance in series with a capacitance of 1–2 μμf, is a difficult and tedious job at microwave frequencies, because we are at the limit of known microwave measurement technique. If, as in a diode, the above impedance is enclosed in a package with parasitic circuit elements, and the signal voltage across the diode should not exceed 30 mv, the measurement problem is still more complicated.

We may look at the interaction problem as shown schematically in Fig. 3(a), where Z_{in} is the measured impedance at the package terminals including the losses in the measurement jig, and Z_j is the diode junction impedance. The circuit designer is mostly concerned with Z_{in} and the device designer with Z_j.

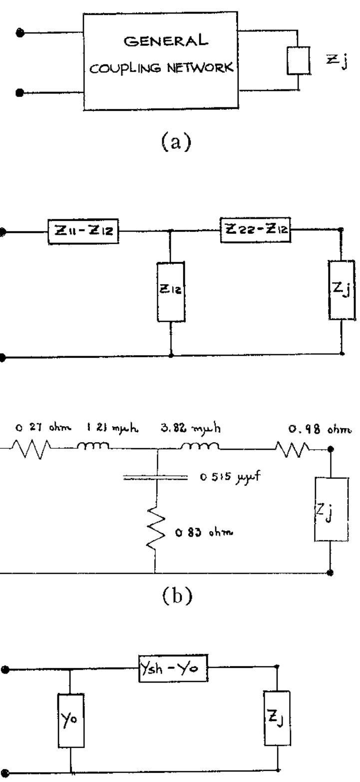


Fig. 3—(a) A four-terminal network representing the transformation from the diode junction impedance to the package terminal or the reference plane; (b) a four-terminal network represented by general Z parameters, and a typical equivalent circuit at 4 kMc; (c) an approximate impedance transformation network which is valid up to 2 kMc.

Approach 1

We can immediately see that the problem of referring the measurements to the active region of the device is one of determining the transformation from the transmission line in which the measurements are made to the interior of the device. The transformation can be represented in a number of ways, the most common of which is the impedance representation

$$V_1 = I_1 Z_{11} + I_2 Z_{12}$$
$$V_2 = I_1 Z_{21} + I_2 Z_{22}$$
$$Z_{12} = Z_{21}, \tag{9}$$

mainly because of its extensive use at lower frequencies. Then we get the equivalent circuit shown in Fig. 3(b) and the measured impedance is

$$Z_{\text{in}} = Z_{11} - \frac{Z_{12}^2}{Z_{22} + Z_j}. \tag{10}$$

Since there are three parameters to be determined, we need, formally, three equations. If special terminations like $Z_j = 0$, ∞, and a standard impedance are used, the equations become simplified (Method A). It should be noted that this transformation will take care of all the defects of the diode holder as well as making corrections for the package losses and reactances.

One of the difficulties now is to make a good standard impedance. The impedance may consist of a small area contact to a resistive material. Assuming a perfectly hemispherical contact, one may easily integrate a spreading resistance of

$$r = \frac{1}{2\pi a \sigma}, \tag{11}$$

where σ is the conductivity of the material, and a is the radius of the contact. *P-P-* or *N-N-*type configuration is desirable. The resistance can be checked out by direct current, low frequencies, and microwave measurements. The diode itself can also be used as an impedance standard (Method B), since we know that the series resistance, R_s, is bias-independent at the higher microwave frequencies, and the reactance variation with bias is known from low-frequency measurements. We then need four instead of three equations to determine our three parameters, Z_{11}, Z_{12}, and Z_{22}. The terminations may then be

$$Z_{j1} = 0$$
$$Z_{j2} = \infty$$
$$Z_{j3} = R_s + jX_1$$
$$Z_{j4} = R_s + jX_2. \tag{12}$$

X_1 and X_2 are the known reactances from low-frequency measurement of the capacitance at two different bias voltages.

From the physical considerations of the diode package and from measurement of the four terminal transformations, a simple transformation from Z_{in} to Z_j has been found to be valid up to 2 kMc (Method C). This simple transformation can be represented as in Fig. 3(c), and we get

$$Z_j = \frac{1}{Y_{\text{in}} - Y_o} - \frac{1}{Y_{sh} - Y_o}, \tag{13}$$

where Y_0 and Y_{sh} are the open-circuited and short-circuited diode admittances. The transformation can easily be made on a Smith Chart, but because of the greater accuracy and ease with which a great number of measurements could be evaluated, the values of Z_j were usually computed using the IBM 704 computer.

Since exploration of the standing-wave pattern is one of the most convenient ways of measuring impedances, the slotted-line technique was used from 300 Mc to 5 kMc. The measurement setup shown in Fig. 4 can measure standing-wave ratio as high as 500. The measurement procedure consists first of finding the value of the voltage minimum. Next, the distance is found between positions at which the output power at the probe is twice the minimum value. Then

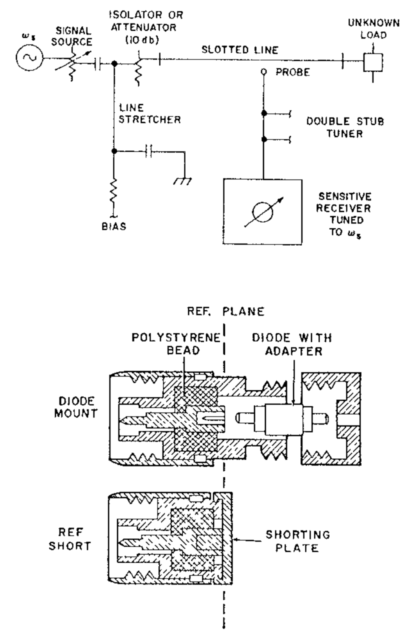


Fig. 4—Experimental measurement setup, used in Approach 1, for measuring very high-standing wave ratios.

$$\text{VSWR} = \rho = \frac{\lambda}{\pi \Delta x}, \tag{14}$$

where λ is the wavelength in the coaxial slotted-line section [2]. The shift in the minimum position, l, when the load is replaced by a short, is also recorded. Then, for instance, in the case of the transformation in (13), the admittance of the open diode package Y_o, the shorted one Y_{sh}, and the actual diode admittance Y_{in}, are computed from

$$Y_{in,o,sh} = \frac{1}{Z_0} \frac{\rho - j \tan \frac{2\pi}{\lambda} l}{1 - j \tan \frac{2\pi}{\lambda} l}, \qquad (15)$$

and substituted in (13). A slotted line with small probe width can be used together with a precision dial indicator as a standard test setup.

Measurement data taken at 2 kMc for a germanium diode junction impedance ($R_s = 5.5$ ohms, $C_T = 2.1$ $\mu\mu$f) using the general transformation Method A with a standard resistor of 15 ohms, according to (11), compared within 5 per cent with results obtained using the diode itself as a standard impedance (Method B). The standard resistors were measured at direct current, low frequency, and 1 kMc, and no noticeable frequency variation was found. Since both theory and measurements predict the same capacitance at microwave frequencies as at low frequency, and since the series resistance in the frequency range above 1 kMc does not show any change with bias voltage, the diode itself seems to be a very reliable standard. In computing the result, Method B is also easier to handle than Method A.

In Table I, Methods B and C are compared for different bias voltages and frequencies at and above 2 kMc. The errors in the results of Method B are estimated to be less than 15 per cent when the diode has a Q of 100 at the operating frequency and bias voltage, and the errors are on the order of 5 per cent for diode Q's of 30 or less. Method B has been used extensively at 4 kMc as a comparison method in our laboratory. Below 2 kMc, earlier measurements showed very good agreement between these two methods. Thus, below 2 kMc, Method C is preferable for laboratory investigations of possible changes of R_s and C_T with, for instance, different surface treatments of the diode and so forth, since no assumption is made with respect to resistance and capacitance changes.

TABLE I

Frequency (kMc)	Bias (volts)	Method B		Method C	
		R_s (ohms)	C_T ($\mu\mu$f)	R_s (ohms)	C_T ($\mu\mu$f)
2	0	6.32	0.787	5.21	0.840
	−1	6.1	0.520	4.79	0.569
3	0	6.55	0.776	4.40	1.045
	−1	6.1	0.522	4.40	0.702
4	0	5.79	0.775	2.60	2.110
	−1	5.27	0.499	2.60	1.36
6	0	5.14	0.80	0.05	—
	−1	5.9	0.531	0.06	—

The low-frequency capacitance at 0 volt = 0.786 $\mu\mu$f; at −1 volt = 0.530 $\mu\mu$f.

Measurements were taken as low as 300 Mc on a gold-bonded germanium diode using Method C. Fig. 5 shows the variation of the equivalent series resistance with frequency and ambients, and the equivalent circuit in Fig. 6 is proposed to explain the variation of the resistance. L is the lead and whisker inductance (4 mμh), C' (0.1 $\mu\mu$f) is the stray capacitance in the package, R_{s1} is the resistance along the surface which shunts the transition capacitance, C_T; and C_s is the capacitance associated with the surface space-charge region. In addition, we have introduced a second surface resistance R_{s2} which expresses the dependence of the reverse current on the surface generation rate. The dc leakage current is mainly determined by R_{s2}, which is in the order of megaohm. At microwave frequencies, C_s effectively short circuits R_{s2} so that the equivalent series resistance can be expressed by

$$R_s' = R_s + \frac{1}{\omega^2 C_T^2 R_{s1}}. \qquad (16)$$

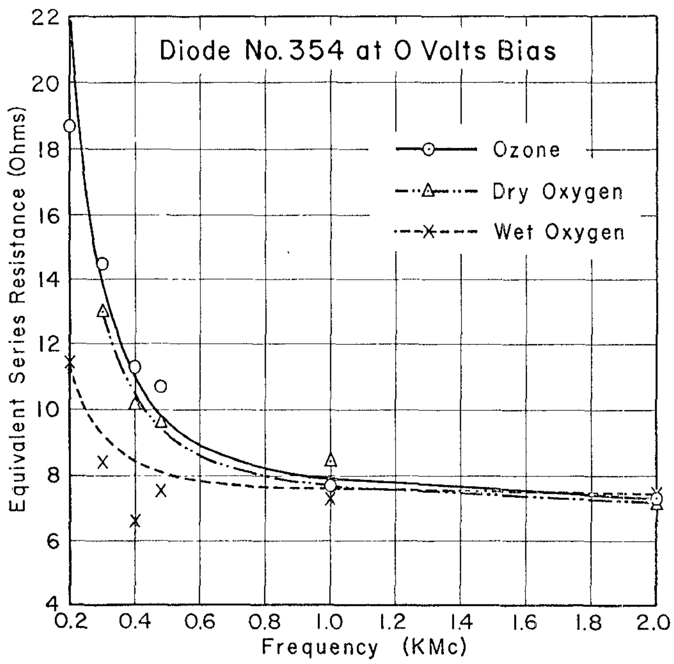


Fig. 5—The equivalent series resistance as a function of frequency and ambient for a Hughes HPA2800 germanium diode.

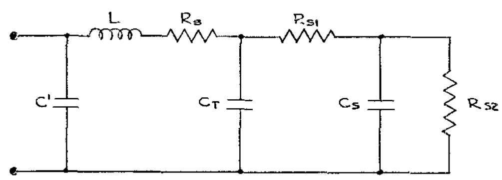


Fig. 6—Proposed equivalent circuit of a gold-bonded germanium diode in the high frequency and lower microwave range.

The frequency dependence on the equivalent series resistance is only pronounced below 500 Mc. More detailed information about this experiment is reported elsewhere [3].

Approach 2

We know that the microwave circuit can be described in a manner that closely resembles low frequency practice. Thus, a description of a cavity resonator from the low frequency point of view is complete if the shunt conductance G_0, together with the angular resonance frequency ω_0, and the Q, Q_0, are given. Since the ordinary low frequency concept of current and voltage, however, does not hold at microwave frequencies, we will define the shunt conductance as

$$G_0 = \frac{2W}{\left[\int E dl\right]^2}, \tag{17}$$

where W is the power dissipated in the cavity and E is the electric field.

The method which will be described to find the Q of the diode will be deduced considering the cavity with input and output terminals used as a transmission device (Fig. 7). The output signal is measured as a function of frequency. If the bandwidth of the receiver is much smaller than the bandwidth of the cavity with and without the diode, the indicator traces a resonant curve from whose bandwidth the Q value of the cavity can be computed. By using a sweep generator as input and a broad-band detection system, we could make a dynamic observation of the cavity characteristic on a scope. Such a system has been made and the performance is very satisfactory. It should be noted that since the voltage across the diode should not exceed 25 mv and the input and output coupling should be very loose, a sensitive detection system is required. The equivalent circuit of the system is shown in Fig. 8(a) for the series resonant case. The cavity resonance is represented by L, C, and R_s; and Z_1 and Z_2 are the self-impedances of the coupling elements. Neglecting the self-impedances, we get a simple equivalent circuit referred to the middle loop as represented in Fig. 8(b). Then the unloaded Q of the cavity

$$Q_0 = \frac{\omega_0 L}{R_s}, \tag{18}$$

and the loaded Q of the system

$$Q_L = \frac{\omega_0 L}{R_s + N_1^2 R_g + N_2^2 R_L}. \tag{19}$$

The input and output coupling coefficients are defined as

$$\beta_1 = N_1^2 \cdot \frac{R_g}{R_s}$$

$$\beta_2 = N_2^2 \cdot \frac{R_L}{R_s}, \tag{20}$$

and we then get,

$$Q_0 = Q_L(1 + \beta_1 + \beta_2). \tag{21}$$

If we now consider the transmission through the cavity as a ratio of the power delivered to a matched load and the maximum power available from the generator, it is noted that the half-power points of transmission occur for

$$Q_L = \frac{\omega}{\Delta\omega}, \tag{22}$$

if the signal generator and detector impedances are both matched. The coupling coefficients should be adjusted to a minimum so that $Q_0 \cong Q_L$. Thus we have tried to obtain a high-Q-cavity system in which the Q of the system can be determined by a measurement of the resonant frequency and the bandwidth.

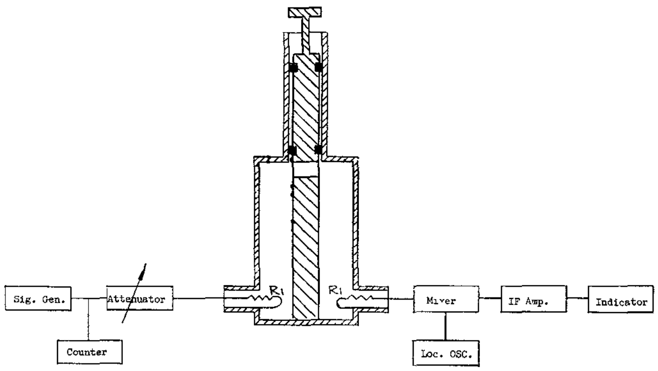


Fig. 7—Schematic diagram showing the measurement setup for the resonant cavity method.

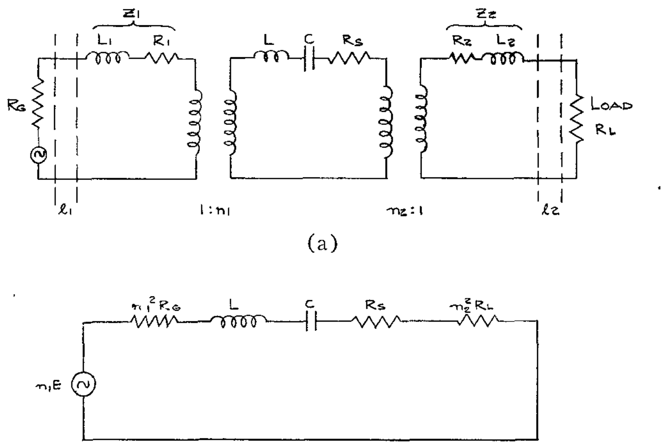


Fig. 8—(a) An equivalent circuit of the measurement system; (b) a reduced equivalent circuit of the measurement system.

We will now assume that the cavity as seen from the diode junction can be represented by a simple parallel resonant circuit. (A series representation may also be valid for cavity characteristics slightly different from those used in the experiment.)

The measuring procedure consists first of substituting an open diode in the cavity in order to include package- and bias-circuit losses and reactances as part of the cavity. Then we get

$$Q_0 = \frac{\omega_0 C_0}{G_0}. \qquad (23)$$

If an arbitrary diode is now inserted in the cavity we get a new Q, Q_m, and a new angular resonance frequency ω_m. Then,

$$Q_m = \frac{\omega_m(C_0 + C_T)}{G_0 + G_s}; \qquad (24)$$

and since $Q_d = (\omega_m C_T)/G_s$ is the real Q of the diode, we get

$$Q_d = \frac{C_T Q_m}{C_0 + C_T - \dfrac{G_0 Q_m}{\omega_m}}, \qquad (25)$$

where Q_m and ω_m are measured quantities and C_T is known from low-frequency measurement of capacitance. If the cavity with the open diode inserted was completely characterized so that C_0 and G_0 were known, (25) would be a simple expression for the diode Q, especially if the cavity were designed so that the effect of the term $(G_0 Q_m)/\omega_m$ is negligible. With this purpose in mind, the cavity in this experiment was designed for a high Q and with a characteristic impedance of 75 ohms. The resonant frequency was chosen to be as low as 1.2 kMc in order to facilitate ease of adjustment of the coupling loop, to escape higher modes, and to enable us to utilize equipment on hand in our laboratory.

The calibration of the cavity was accomplished by two different procedures. Procedure A makes use of the measured changes in resonance frequency of the cavity by a known amount of change in the diode capacitance according to

$$\left(\frac{\omega_1}{\omega_2}\right)^2 = \frac{C_{T_2} + C_0}{C_{T_1} + C_0}, \qquad (26)$$

from which C_0 can be found. Then G_0 can be calculated from (23). Procedure B uses a standard admittance diode, $G_s + j\omega C_{T_s}$, measured as in Approach 1, Method B. With this diode inserted in the cavity, we have

$$Q_s = \frac{\omega_s(C_0 + C_{T_s})}{G_0 + G_s}, \qquad (27)$$

which, combined with (23) gives

$$G_0 = \frac{G_s - \omega_s \dfrac{C_{T_s}}{Q_s}}{\dfrac{Q_0 \omega_s}{Q_s \omega_0} - 1}; \qquad (28)$$

and C_0 can be found from (23). Table II shows the results obtained from both calibration procedures. Measurements on several diodes indicate the same behavior. Since Procedure A is independent of knowing the value of a standard impedance, we found it reasonable to use an average value of 1.1 $\mu\mu$f for C_0 and 11 μmhos for G_0, according to the table.

Using this cavity resonance method in measurements on several Hughes silicon diodes (25), the results in Table III were obtained. A comparison with Method B in Approach 1 is also shown.

TABLE II

Bias (volts)	C_T ($\mu\mu$f)	Q	Procedure A			Procedure B		
			$\omega/2\pi$ (Mc)	C_0 ($\mu\mu$f)	G_0 (μmhos)	G_s (μmhos)	G_0 (μmhos)	C_0 ($\mu\mu$f)
+.3	0.9	94	895	1.19	11.9	108	11	1.1
0	0.68	130	946	1.1	11	69	10.7	1.07
−1	0.48	182	1005	1.0	10	39	9.1	0.91
−4	0.34	340	1056			21.5	15.8	1.52
Open diode		750	1200					

TABLE III

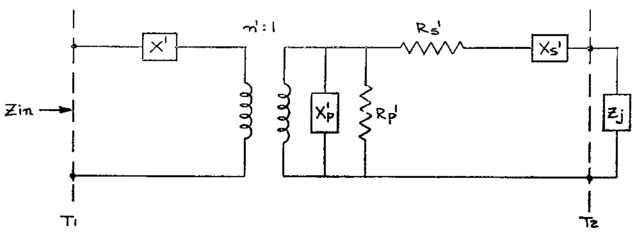
Diode Number	Approach 1, Method B f_c (kMc)	Approach 2 f_c (kMc)
SM1-AA-2	52.7	50
SM1-AA-4	50.7	46.5
1008	54.9	50
1009	12.4	11.6
1011	32.2	36.2
1015	45.8	44.2

It can be concluded that the cavity resonance method is fairly accurate, and, if a sweeping technique was utilized, the method would be extremely useful for routine tests of diode cutoff frequency in the laboratory as well as in production.

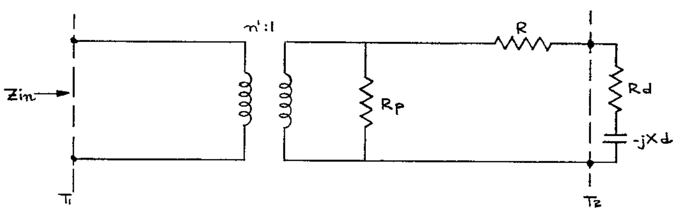
Approach 3

A general four-terminal coupling network can, at a particular frequency, be represented by a canonical network postulated by Weissfloch [1]. The equivalent circuit [Fig. 9(a)] is represented by a lossy and lossless part between the reference planes T_1 and T_2, which define the ends of the physical structure representing the transformation.

We will now add stub tuners to the transformation between planes T_1 and T_2 so that when open and short circuits are substituted for the diode junction Z_j, the position of the minimum of the standing waves in the slotted line is shifted by a quarter wavelength. The reactive elements of the transformation have been tuned out and, with a precision package and diode holder, the equivalent circuit of the transformation will now be as shown in Fig. 9(b). The impedance transformation n^2/Z_0, the series resistance R, and the shunt resistance R_p, can be determined by measuring Z_{in} when Z_j is represented by a known standard capacitance, a short circuit, and an open circuit. The reference position for the minimum of the standing-wave pattern is established for $Z_j=0$ or ∞. Using the diode capacitance C_s, measured at low frequency as our standard capacitance, and when a measurement frequency is chosen so the effect of R_p is very small, we get



(a)



(b)

Fig. 9—(a) A general Weissfloch canonical network; (b) a simplification of the canonical network.

$$Z_{in_d} = R_{in_d} - jX_{in_d} = \frac{n^2}{Z_0}(R + R_s) - j\frac{n^2}{Z_0}X_s, \quad (29)$$

which gives

$$\frac{n^2}{Z_0} = \frac{X_{in_d}}{X_s}, \quad (30)$$

and from the open- and short-circuit measurements

$$R = \frac{Z_0}{n^2} R_{in_{sh}} \quad (31)$$

$$R_p = \frac{Z_0}{n^2} R_{in_{sh}}. \quad (32)$$

The measurement system is now fairly well calibrated, and we can write for the Q of the diode in terms of the measured Q, Q_m, of the input impedance and the equivalent circuit parameters

$$Q_d = Q_m\left(1 + \frac{R}{R_d} + \frac{(R + R_d)^2 + X_d^2}{R_pR_d}\right). \quad (33)$$

A measurement frequency was chosen high enough so that the third term in this expression is very small. In order to get an approximate idea of the correction term within the parenthesis at 9 kMc, it should be mentioned that the standing-wave ratio was in the order of 100 for an open diode and 10 for a shorted one. If we now take a silicon diode like SM1-AA-2 (see Table IV), with $n^2/Z_0=0.1$, we get $R_p=1000$ ohms and $R=1$ ohm, and with a normalized input impedance of 0.5–j2.15 (see Table IV), and zero-bias capacitance of 0.82 $\mu\mu$f, we have at 9 kMc $X_d=21.5$ ohms and $R_d=4$ ohms, which gives

$$Q_d = Q_m(1 + 0.25 + 0.12) = \frac{f_c}{f}\cdot \quad (34)$$

Thus, if Q_m is taken directly from the Smith Chart, the cutoff frequency of this diode should be in the order of 37 per cent higher than the cutoff frequency calculated from Q_m.

TABLE IV

Unit	C_0 ($\mu\mu$f)	VSWR	Phase Shift (Fractional Wavelength from Short)	Z_{in_d} Normalized	Q_m	f_c (kMc) (Correction Applied)	n^2/Z_0
SM-AA-1	0.69	13.0	0.169	0.60–j2.40	4.0	49.2	0.09
AA-2	0.82	13.0	0.183	0.50–j2.15	4.3	53.0	0.1
AA-3	0.37	18.0	0.210	0.85–j3.7	4.4	54.2	0.08
AA-4	0.95	9.5	0.166	0.39–j1.65	4.2	51.6	0.09

A block diagram of the equipment used in several measurements on Hughes silicon diodes is given in Fig. 10. The E-H tuner was adjusted according to two requirements. There must be a quarter-wave shift between open and shorted diodes and, further, the transformation ratio, n, must be such that Z_{in} can be plotted easily on a Smith Chart. The following procedure was found to be convenient:

a) match a diode with tuners and sliding short,
b) replace diode with open-circuit diode and note position of minimum,
c) insert shorted diode and adjust sliding short to get quarter-wave shift in the minimum of the standing-wave pattern.

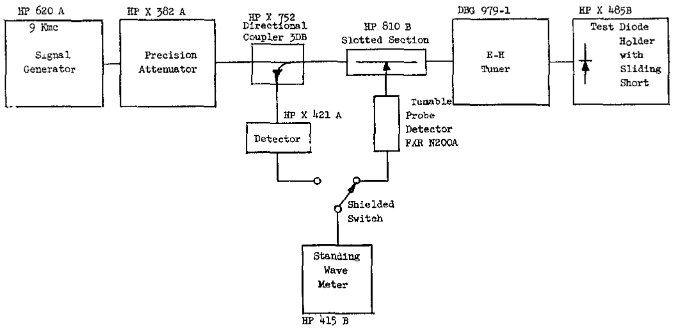


Fig. 10—Schematic diagram showing the measurement setup for the X-band measurement according to Approach 3.

Step a) is not absolutely necessary and it will be invalidated when the sliding short is readjusted in c). However, with most diodes tested, this procedure gave a usable impedance transformation without numerous readjustments of the tuners and the sliding short. After these adjustments, routine testing of diodes can be accomplished without retuning the elements.

In Table IV are the results of measurements on four silicon diodes. The measured impedances of the four diodes are plotted in Fig. 11, on page 20 using data from the table. These data are self-consistent in the respect that the measured reactances are nearly in the same proportion among units as the reactances calculated from low frequency C_0's. This is indicated in the table by the calculated n^2/Z_0 for each diode, which should be the same for all diodes, because the tuners were not touched during the set of measurements.

Fig. 12 on page 21 shows the measured impedances of a diode for various dc bias. According to our equivalent circuit, the points should fall on a circle of constant resistance, at least for the reverse and low forward biases. This plot supports the validity of the method. The points here are nearly on the circle of $R/Z_0=1$, which was only a coincidence in this particular experiment. However, as mentioned above, it was intended to transform the impedance to any convenient place in the middle of the Smith Chart. Also plotted are the impedances of four shorted packages with short no. 11 as the phase reference. (It was also used as reference in plotting the diode impedance.) The reactance variation among shorted packages is about 3 ohms. Thus, it appears that the package is not quite good enough and causes errors from the assumption of identical packages. Several open circuit packages were measured, and they all had practically the same phase. The shorts were made by pressing the contacting posts against the diaphragms, and the opens were made by first contacting and then backing off a very slight amount.

Finally, in Table V is a comparison of the different approaches for five Hughes Silicon mesa diodes. The cutoff frequency is defined at zero-volt bias. Approach 3 is similar to Waltz's method [4]. The only difference is in the calibration procedure and that R_p is included in our equivalent circuit. The same measurement setup as in Fig. 10 was also used for Houlding's method [5]. The result is shown in the table to make the comparison as complete as possible. Since it has been experimentally verified that the loss resistance of the diode is also constant above 500 Mc, it can be concluded from the table and also from a large number of other measurements that the last two methods in the table sometimes can show a considerable error. The correction applied in Approach 3 is made according to (33).

TABLE V

Diode Number	Approach 1, Method B, at 4 kMc f_c (kMc)	Approach 2 Around 1 kMc f_c (kMc)	Approach 3 at 9 kMc, (Correction Applied) f_c (kMc)	Houlding's Method [5] (No Correction Applied) f_c (kMc)
SM1-AA-2	52.7	50	53.0	47.3
SM1-AA-4	50.7	46.5	51.6	44
1009	12.4	11.6	11.5	11
1011	32.2	36.2	10.0	14.2
1015	45.8	44.2	25.6	39.3

In the method used by Houlding [5], the effect of the parallel resistance R_p is neglected and the series resistance R is made part of the diode losses. Thus the source of error in this method is the circuit losses. Since both these two resistances are varying with the position of the E-H tuner and the sliding short settings, the measurements are also not quite repeatable when the tuner positions are changed. Thus, if the method is calibrated, the calibration will change from the measurement of one diode to another. Since each diode is matched individually in this method, the effect of variations in the diode packages is compensated for.

The reason for the discrepancy in f_c of diode 1011 and 1015 in Approach 3, compared with Approaches 1 and 2, is the fairly large diode capacitance of these two diodes compared with the other three diodes (C_T of SM1-AA-2 is 0.82 $\mu\mu$f and C_T of 1011 is 3.14 $\mu\mu$f). The method assumed identical shorted packages. However, since the reactance variation among shorted packages

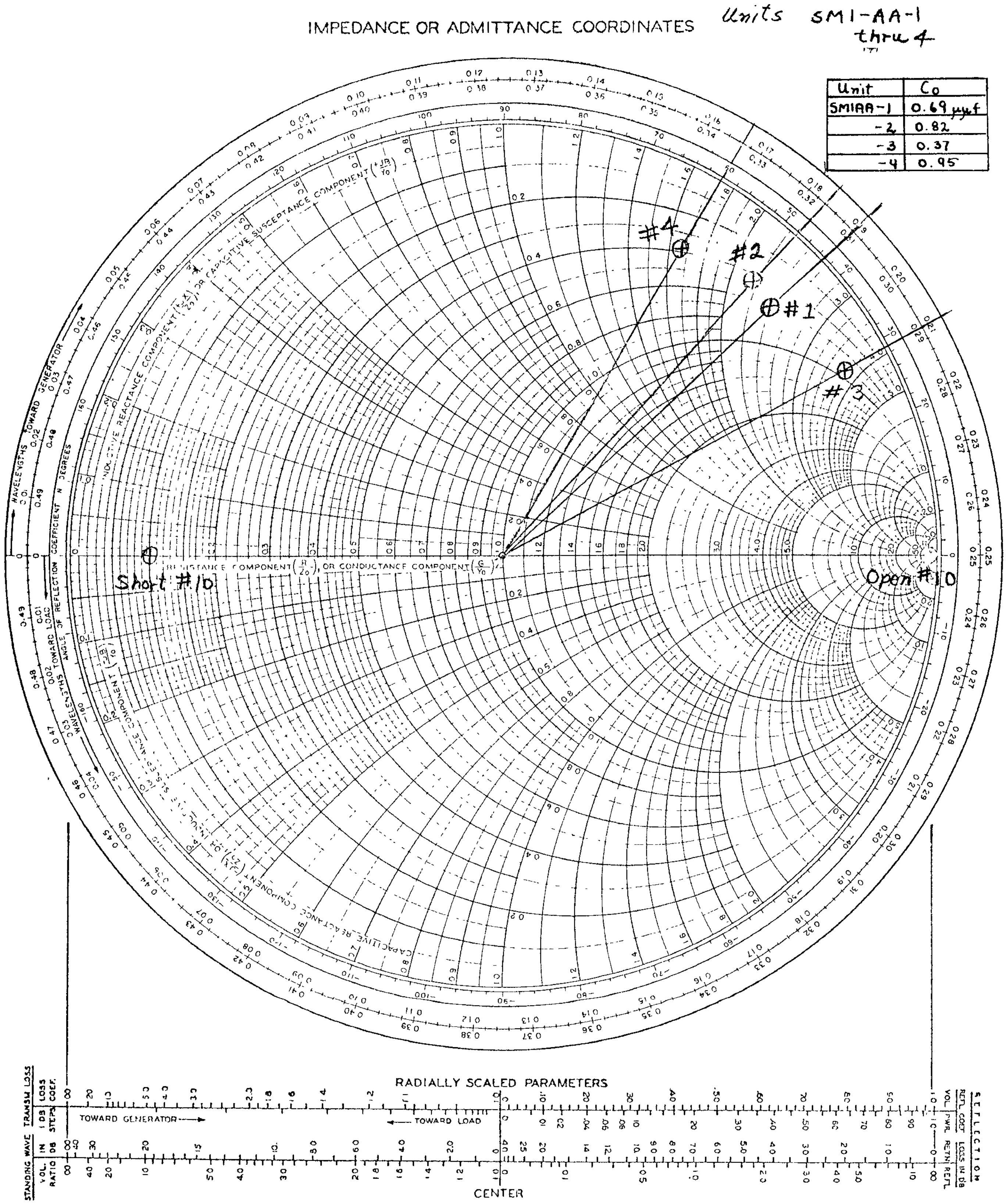
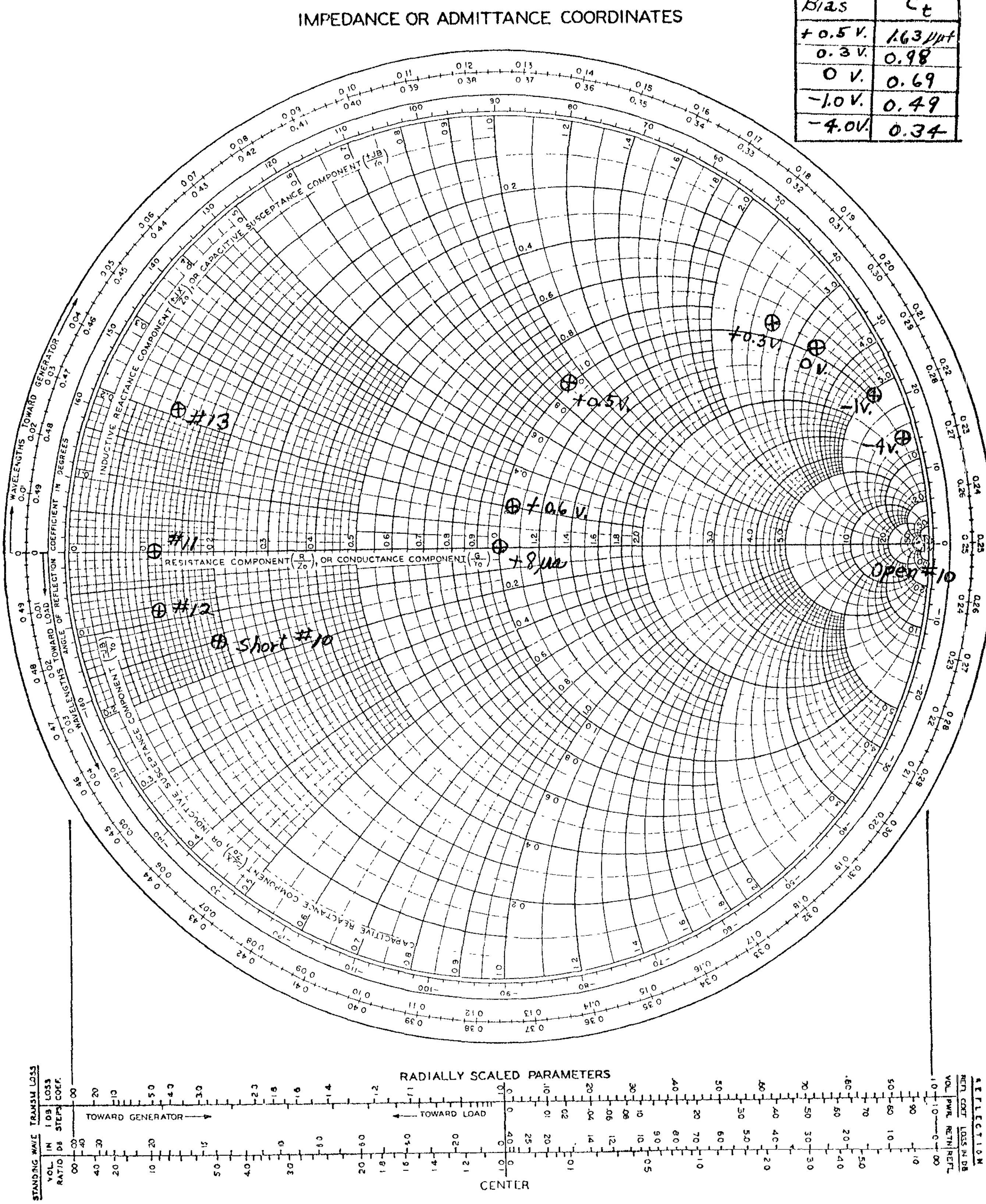


Fig. 11—The microwave impedance of four diodes plotted on the Smith Chart.



Bias	C_t
+0.5 V.	1.63 μμf
0.3 V.	0.98
0 V.	0.69
-1.0 V.	0.49
-4.0 V.	0.34

Fig. 12—The microwave impedance variation with bias voltage plotted on the Smith Chart.

can vary several ohms at 9 kMc (Fig. 12), the error in f_c for these high capacitance units can easily be understood.

Of the first two approaches which have given consistent results, the cavity-resonance method is by far the simplest and most adapted for very high Q diodes. The method is easy to calibrate and the measurement is not affected by variations in the diode packages, since the measurement frequency is chosen as low as 1 kMc. In Approach 3, if the diode Q is higher than 5 at 9 kMc, then a higher frequency of measurement is recommended in order to be able to neglect R_p in our equivalent circuit. A higher measurement frequency, however, requires extreme precision in package, in diode holder and in all respects of the measurements.

V. Conclusions

A general representation has been given of the importance and requirements of the fundamental electrical parameters of microwave variable capacitance diodes. It is suggested that the following parameters be specified:

1) the breakdown voltage,
2) the capacitance at zero-volt bias,
3) the exact variation of the capacitance with bias voltage (from which the nonlinearity coefficient can be calculated),
4) the cutoff frequency or the Q at a particular frequency,
5) the dynamic back-biased resistance (because the diodes also have applications at the lower frequencies).

The Q of the diode is not an easy parameter to measure; therefore, three different approaches were considered in order to gain comparison. In the first method, a general four-terminal transformation is used from the active region of the device to the transmission line in which the measurements are made. Special terminations such as open circuit, a short circuit, and a standard impedance were used instead of the active diode region when determining the general four-terminal coupling network. It was shown that a simple L network was a good approximation up to 2 kMc, and it led to a fairly accurate and easy method for measuring the diode junction impedance. In the second approach, the diode Q was deduced by placing the diode in a cavity resonator which is considered as a transmission device. From the measured bandwidth and the resonant frequency of the cavity with the diode inserted, the diode Q could be computed after the cavity, without the diode, was first calibrated by using a known capacitance change. Finally, a simplified form of a general Weissfloch canonical network was presented, which led to another Q-evaluation method.

The results obtained from all these methods compare fairly well. However, from measurement on several diodes, it can be concluded that the cavity resonance method (at around 2 kMc) may be the best for a specification test because of the ease with which these measurements can be accomplished, and the accuracy and the reproducibility of the results. In order to standardize this method, the cavity dimensions have to be specified for such a test.

VI. Acknowledgment

The author wishes to thank D. English and R. Knox for their significant help in performing the measurements.

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Correction

W. H. Eggiman, author of "Scattering of a Plane Wave on a Ferrite Cylinder and Normal Incidence," which appeared on pp. 441–445 of the July, 1960, issue of these Transactions, has brought the following to the attention of the *Editor*. By an oversight, the sponsoring agency of the work described in the paper was not mentioned. The author regrets the omission very much and wishes to express his sincere appreciation to the Electronics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command, which supported the work under contract No. AF 19 (604)-3887.